MATH 1A - MOCK MIDTERM 3

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Name: _____

Instructions: This is a mock midterm, designed to give you an idea of what the actual midterm will look like. Make sure you do it, the actual exam will be very similar to this one (in length and in difficulty)! Remember that this midterm is 2 hours long!

Warning: The questions on the actual midterm may look completely different from the questions on this mock midterm, although the basic structure will be the same, so make sure to study the problems from the related rates and optimization handouts as well!

1		25
2		10
3		15
4		10
5		10
6		10
7		20
Bonus 1		5
Bonus 2		5
Total		100

Date: Friday, July 22nd, 2011.

1. *(25 points)* Sketch a graph of the function $f(x) = e^{\frac{-x^2}{2}}$. Your work should include:
   - Domain
   - Intercepts
   - Symmetry
   - Asymptotes (no Slant asymptotes, though)
   - Intervals of increase/decrease/local max/min
   - Concavity and inflection points

   Note: This function is used a lot in statistics, it is called the **normal distribution function**.

(This page is left blank in case you need more space to do question 1.)

2. *(10 points)* Use linear approximations (or differentials) to find an approximate value of $(3.01)^3$

3. *(15 points)* Two people start moving from the same point. One person travels north at a speed of 3 mph and the other person travels east at a speed of 4 mph. At what rate is the distance between the two people changing after 2 hours?

4. *(10 points)* Evaluate the following limits:

(a) $\lim_{x\to 0} \frac{x+\sin(x)}{1+\cos(x)}$

(b) $\lim_{x\to -\infty} xe^x$

(c) $\lim_{x\to 0} x^x$

5. *(10 points)* Find the absolute maximum and minimum of $f(x) = x^3 - 3x$ on $[0, 2]$

6. *(10 points)* Suppose f is an odd function and is differentiable everywhere. Let b be given. Show that there is a number c in $(-b, b)$ such that $f'(c) = \frac{f(b)}{b}$

   Hint: Let $a = -b$

7. *(20 points)* Find the point on the line $x + y = 1$ that is closest to the point $(-3, 1)$.

Bonus 1 *(5 points)* Use l'Hopital's rule to show:

$$\lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x)$$

Note: Careful! You're differentiating with respect to h here, not x !

Bonus 2 *(5 points)* (Courtesy Adi Adiredja)

Omar didn't get *the* girl. Sad and frustrated he decided to run around the track at the gym. When he got to the track, one girl who was about to start running captured his attention. Omar hurried so they could start running together. They started at the same time, but then she started running faster. Omar sped up, but then she passed him again. Next thing he knew, the two of them were racing. At some point she noticed the finish line, and yelled out,"I dont go for losers!" Omar ran as fast as he could and the race ended in a tie. After the race she confidently came up to him and said, "I dont *mean* to be rude, but I only *value* smart guys. If you can prove the next thing I say, I'll go on a date with you."

She then said, "We started the race at the same time, and the race ended in a tie, I claim that at some point during the race we were running at the same speed." She smiled at Omar and said, "Are you smart enough?" Use Calculus to help Omar!

Hint: Let $g(t)$ and $h(t)$ be the position functions of the two runners, and consider $f(t) = g(t) - h(t)$